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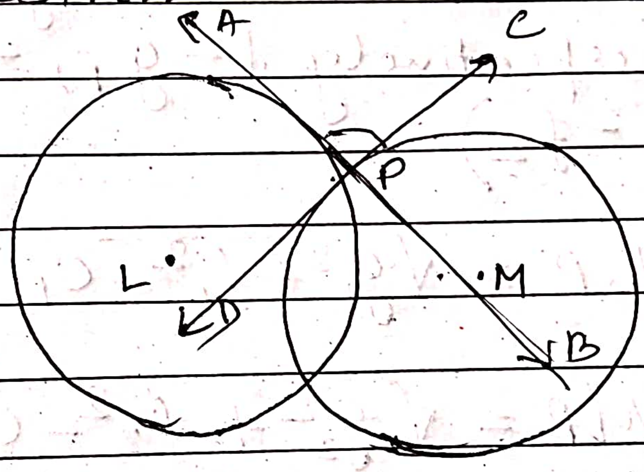
Definition of angle of intersection of two circles

By the angle of intersection of two circle we mean the angle between their tangents at the common point of intersection.



In the adjoining fig AB & CD are tangents to the circles at the common point P of intersection.
Then the angle APC or CPB is the angle of intersection of two circles.

To find the angle of intersection of two circles.

Let the two circles given by

$$x^2+y^2+2g_1x+2f_1y+c_1=0$$

and $x^2+y^2+2g_2x+2f_2y+c_2=0$

intersect at P, the tangents at which are PT_1 and PT_2 respectively. Let $(T_2PT_1$

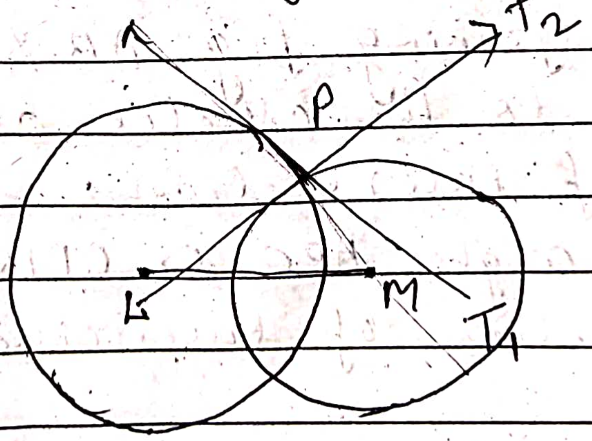
~~Let L & M be~~

Let $\angle T_2PT_1 = \theta$

Let L & M be the centres of the two circles. Join L-P, L-M and P-M. Then the co-ordinates of L and M are respectively $(-g_1, -f_1)$ and $(-g_2, -f_2)$.

Also $LP = \sqrt{g_1^2+f_1^2-c_1}$

and $MP = \sqrt{g_2^2+f_2^2-c_2}$



$$LM^2 = (g_1-g_2)^2+(f_1-f_2)^2$$

We know that the angle between any two lines is equal to the angle between their normals

Evidently $LP \perp PT_1$ and $MP \perp PT_2$.

$\angle LPM = \angle T_2PT_1 = \theta$ or $\pi - \theta$

From ΔLPM, we have

$$\pm \cos\theta = \frac{LP^2 + MP^2 - LM^2}{2\,LP \cdot MP}$$

$$\cancel{\pm \cos\theta = (g_1^2 + f_1^2 - c_1) + (g_2^2 + f_2^2 - c_2)}$$

$$\cancel{- [fg}$$

$$\pm \cos\theta = \frac{(g_1^2 + f_1^2 - c_1) + (g_2^2 + f_2^2 - c_2) - [(g_1 - g_2)^2 + (f_1 - f_2)^2]}{2\sqrt{g_1^2 + f_1^2 - c_1} \cdot \sqrt{g_2^2 + f_2^2 - c_2}}$$

or, $$\cos\theta = \pm \frac{2g_1g_2 + 2f_1f_2 - c_1 - c_2}{2\sqrt{g_1^2 + f_1^2 - c_1} \cdot \sqrt{g_2^2 + f_2^2 - c_2}}$$

This gives the required angle.